

A Note on Bounds of Scalar Operators in Perturbative SCFTs

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Abstract

Bounds on anomalous dimensions of scalar operators in 4d superconformal field theory are explored through perturbative viewpoint. Following the recent work of Green and Shih, in which a conjecture involved this issue is verified at the NLO, we consider the NNLO corrections to the bounds, which are important in some situations and can be divided into two cases where $\mathcal{O}(\lambda^4)$ or $\mathcal{O}(y^2)$ effects dominate respectively. In the former case, we find that the conjecture is maintained at NNLO, while in the later case the statement still holds due to null correction.

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1 Introduction

Conformal field theory (CFT) (see [1, 2, 3] for example), which is tied to important concepts in field theory and phenomenological application, has been extensively explored. For example, the small hierarchy μ problem involved in electroweak symmetry breaking in the minimal supersymmetric standard model can be solved when this theory is coupled to a *hidden* superconformal field theory (SCFT) [5, 6] (for other recipes, see [4] and reference therein for example). The reason for this viability is due to the different scaling behaviors between chiral μ and real B_μ operator, which is expected in SCFTs where the condition $\delta_{min} > 0$ (see its definition in (1.4)) is satisfied.

Given a CFT, the dimensions of operators and coefficients in the correlator functions (or equivalently the OPE coefficients) of these operators exactly *determine* or *define* the theory. Many efforts have been done by using arguments of conformal symmetry, crossing symmetry and unitarity. Among these developments, an interesting and well-known topic in unitary CFT is the discovery of bounds on dimensions of operators. The full list of unitary bounds, which includes fields with Lorentz spin (j, \tilde{j}) is presented in [1]. Also, a-maximization [8] that follows from the arguments involved in anomalies of global symmetries provides, in terms of unitary constraints, an alternative method to determine the dimensions of chiral operators in SCFTs.

Very recently the bounds on anomalous dimension of primary scalar operators are addressed [11, 12, 13, 14, 15, 16, 17] by applying conformal blocks [9] and global symmetries to exploring the four-point correlators of scalar primary operators. A conjecture is hinted by these works.

The 4d interacting SCFT \mathcal{P}_1 we are going to study contains a chiral operator \mathcal{O} of dimension $\Delta_{\mathcal{O}} = 2 - \epsilon$. The OPE of \mathcal{O} and its anti-chiral field \mathcal{O}^\dagger is assumed to be

$$\mathcal{O}(x)^\dagger \mathcal{O}(0) = \frac{1}{|x|^{2\Delta_{\mathcal{O}}}} + \sum_i \frac{c_i}{|x|^{2\Delta_{\mathcal{O}} - \Delta_i}} L_i + \cdots \quad (1.1)$$

where L_i are real scalar multiplets with dimension $\Delta_i = 2 + \nu_i$ (Here ν_i is a non-negative real number). c_i refer to the OPE coefficients. The terms ignored in (1.1) denote descendants with higher spin. We follow the convention in [10] where all primary scaling operators are canonically normalized as in (1.1). We explore theories constructed through deforming \mathcal{P}_1 by ,

$$\mathcal{L} = \mathcal{L}_{\mathcal{P}_1} + \left(\frac{1}{4\pi^2} \int d^4\theta X^\dagger X + \int d^2\theta \frac{\lambda}{2\pi} X \mathcal{O} + h.c \right) \quad (1.2)$$

with X being a free chiral superfield. Our concern is to discuss the anomalous dimensions of scalar primary operators \mathcal{S}_i which appear in the OPE of X and X^\dagger ,

$$X^\dagger(x)X(0) = \frac{1}{|x|^{2\Delta_X}} + \sum_i \frac{c_i}{|x|^{2\Delta_X - \Delta_i}} \mathcal{S}_i + \cdots, \quad (1.3)$$

When the anomalous dimension of $\Delta_X = 1 + \epsilon$ is small, $0 < \epsilon \ll 1$ as we assume throughout this paper, the deformed theory (1.2) will renormalization group (RG) flow into a new interacting CFT \mathcal{P}_2 . As expected, the candidate operators \mathcal{S}_i in (1.3) include $X^\dagger X$, L_i and their mixing. A variety of works [11, 12, 13, 14, 15, 16, 17] tend to claim that the sign of δ_{min} defined as ¹

$$\delta_{min} = \min(\Delta_i) - 2\Delta_X < 0 \quad (1.4)$$

always holds in general.

The purpose in this article is to study the higher-order corrections on this conjecture in the context of perturbative CFT, by following the method of calculations proposed by Green and Shih [10]. The advantage of this method is that the RG flow between the new and old fixed points is manifest. By using this method, the conjecture is perturbatively verified at the next-to-leading order (NLO). We would like to address the question whether the bound on δ_{min} is robust as suggested. If not, then under which circumstances it can be violated. As we will claim, despite smaller than NLO ones, the NNLO corrections are important and even substantial in some circumstances. In particular, the modifications to the vanishing matrix elements of anomalous dimension of \mathcal{S}_i at NLO can directly affect the sign of δ_{min} , even though they don't substantially modify the values of fixed points couplings λ_* and y_{i*} .

In section 2, we divide the discussions into two cases. In the case where $\mathcal{O}(\lambda^4)$ dominates, we calculate the corrections to values of couplings at the new fixed points in section 3, and estimate the modification to the matrix of anomalous dimension and value of δ_{min} , which are found to be substantial, however, not enough to violate the conjecture. In section 4, we consider the modification due to $\mathcal{O}(y^2)$ effects at NNLO, which is found to be actually *null*. We claim that this observation *exactly* holds beyond NLO. Finally, we summarize our results in section 5.

¹As mentioned in the previous discussion, the study of this conjecture is of interest from point of view of phenomenology.

2 NNLO Corrections

Take the RG effects into account, the Lagrangian for \mathcal{P}_2 SCFT can be written as,

$$\mathcal{L} = \mathcal{L}_{\mathcal{P}_1} + \frac{1}{4\pi^2} \int d^4\theta (1 + \delta Z_X) X^\dagger X + \int d^4\theta (y_i + \delta y_i) L_i + \left(\int d^2\theta \frac{\lambda}{2\pi} \Lambda^\epsilon X \mathcal{O} + h.c \right) \quad (2.1)$$

where we have introduced Λ dependence so that λ is a dimensionless coupling. y_i are the coupling constants appearing in L_i operators. δZ_X and δy_i denote the effects of wave-function renormalization. By using the holomorphic arguments, we find the beta function for λ is exactly given by,

$$\beta_\lambda = -\epsilon\lambda + \lambda\gamma_X(\lambda, y_i), \quad \gamma_X = -\frac{1}{2} \frac{\partial \delta Z_X}{\partial \log \Lambda} \quad (2.2)$$

Expanding the wave-function renormalization functionals δZ_X and δy_i in power of λ and y_i which are both assumed to be small as,

$$\begin{aligned} \delta Z_X &= a_1 \lambda^2 + a_{1i} y_i + a_{2i} \lambda^2 y_i + a_2 \lambda^4 + a_{2ij} y_i y_j + \mathcal{O}(\lambda^6, y^4, \lambda^4 y^2) \\ \delta y_i &= b_{1i} \lambda^2 + b_{1ij} y_j + b_{2ij} \lambda^2 y_j + b_{2i} \lambda^4 + b_{2ijk} y_j y_k + \mathcal{O}(\lambda^6, y^4, \lambda^4 y^2) \end{aligned} \quad (2.3)$$

where a_i, b_i are real coefficients, some of which have been considered in [10] up to NLO,

$$\begin{aligned} a_1 &= \frac{\pi^2}{\epsilon}, \\ a_{1i} &= 0, \\ a_{2i} &= \frac{8\pi^4 c_i}{\nu_i - 2\epsilon} \mathcal{I}(\nu_i, \epsilon), \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} b_{1i} &= \frac{c_i}{2(2\epsilon + \nu_i)}, \\ b_{1ij} &= 0, \\ b_{2ij} &= 0, \end{aligned} \quad (2.5)$$

In the following we take into account the NNLO corrections. In terms of the assumption in (2.3) we can write the beta function of λ and y_i as,

$$\begin{aligned} \beta_\lambda &= -\epsilon\lambda + \lambda \left[\pi^2 \lambda^2 - 4\pi^4 \sum_i c_i y_i \mathcal{I}(\nu_i, \epsilon) \lambda^2 + 2\epsilon a_2 \lambda^4 - \sum_{i,j} a_{2ij} (\nu_i + \nu_j) y_i y_j \right] \\ \beta_{y_i} &= \nu_i y_i - \frac{1}{2} c_i \lambda^2 - (4\epsilon + \nu_i) b_{2i} \lambda^4 + \sum_{j,k} b_{2ijk} (\nu_j + \nu_k - \nu_i) y_j y_k \end{aligned} \quad (2.6)$$

which implies the values of couplings λ_* and y_{i*} at the fixed point of \mathcal{P}_2 ,

$$\begin{aligned}
-\epsilon + \pi^2 \lambda_*^2 - 4\pi^4 \sum_i c_i y_{i*} \mathcal{I}(\nu_i, \epsilon) \lambda_*^2 + 2\epsilon a_2 \lambda_*^4 - \sum_{i,j} a_{2ij} (\nu_i + \nu_j) y_{*i} y_{*j} &= 0 \\
\nu_i y_{i*} - \frac{1}{2} c_i \lambda_*^2 - (4\epsilon + \nu_i) b_{2i} \lambda_*^4 + \sum_{j,k} b_{2ijk} (\nu_j + \nu_k - \nu_i) y_{*j} y_{*k} &= 0
\end{aligned} \tag{2.7}$$

A natural question we have not addressed is under which condition the approximation up to NNLO is important and sufficient, especially in compared with the NLO ones. For corrections to the second equation in (2.7), $y_{i*} \simeq \frac{1}{2} \frac{c_i}{\nu_i} \lambda_*^2$ [10] is always valid except that the new theory \mathcal{P}_2 is beyond the scope of perturbation. This suggests $y_{i*} \ll \lambda_*^2$ if $c_i \ll \nu_i$, or equivalently $c_i \ll 1$, which implies that the effect of $\mathcal{O}(y^2)$ (even of $\mathcal{O}(\lambda^2 y)$) is smaller in compared with that of $\mathcal{O}(\lambda^4)$. It is necessary to take the order of $\mathcal{O}(\lambda^4)$ into account and revise those discussions based on orders up to $\mathcal{O}(\lambda^2 y)$ but without $\mathcal{O}(\lambda^4)$, even though there exists no large hierarchy in the OPE coefficients. Nevertheless, $y_{i*} > \lambda^2$ if $c_i \sim \mathcal{O}(1)$. In this case the corrections arising from $\mathcal{O}(y^2)$ and $\mathcal{O}(\lambda^2 y)$ dominate over $\mathcal{O}(\lambda^4)$.

3 SCFTs at $\mathcal{O}(\lambda^4)$

We perform the perturbative calculations by using the OPEs in appendix A. The rational is that correlation functions must be independent of Λ scale, which results in the requirement that the coefficients appearing in the same operator that carries Λ factor must cancel out. Doing so we obtain,

$$\begin{aligned}
a_2 &= 16\pi^4 \left(\frac{c_i^2}{\nu_i^2 - 4\epsilon^2} \right) \mathcal{I}(\nu_i, \epsilon) - \frac{2\pi^2}{\epsilon^2} \mathcal{T}(\epsilon) \\
b_{2i} &= -\frac{\pi^2 c_i}{2\epsilon(\nu_i - 2\epsilon)} [\mathcal{P}(\nu_i, \epsilon) + \mathcal{Q}(\nu_i, \epsilon)]
\end{aligned} \tag{3.1}$$

where $\mathcal{I}(\nu_i, \epsilon)$, $\mathcal{T}(\epsilon)$, $\mathcal{P}(\nu_i, \epsilon)$ and $\mathcal{Q}(\nu_i, \epsilon)$ are all dimensionless and smooth functionals as defined in appendix A.

Substituting (3.1) into (2.6) and (2.7) while neglecting the $\mathcal{O}(y^2)$ effects results in,

$$\begin{aligned}
\beta_\lambda &= -\epsilon \lambda + \lambda \left[\pi^2 \lambda^2 - 4\pi^4 \sum_i c_i y_i \mathcal{I}(\nu_i, \epsilon) \lambda^2 + 2\epsilon a_2 \lambda^4 + \dots \right] \\
\beta_{y_i} &= \nu_i y_i - \frac{1}{2} c_i \lambda^2 - (4\epsilon + \nu_i) b_{2i} \lambda^4 + \dots
\end{aligned} \tag{3.2}$$

and consequently

$$\begin{aligned} -\epsilon + \pi^2 \lambda_*^2 - 4\pi^4 \sum_i c_i y_{i*} \mathcal{I}(\nu_i, \epsilon) \lambda_*^2 + 2\epsilon a_2 \lambda_*^4 &= 0 \\ \nu_i y_{i*} - \frac{1}{2} c_i \lambda_*^2 - (4\epsilon + \nu_i) b_{2i} \lambda_*^4 &= 0 \end{aligned} \quad (3.3)$$

, respectively. The value of y_{i*} is instead of,

$$y_{i*} = \frac{c_i}{2\nu_i} \lambda_*^2 \left[1 + \epsilon^{-1} \lambda_*^2 (\mathcal{O}(1) + \kappa (\mathcal{P}(\nu_i, \epsilon) + \mathcal{Q}(\nu_i, \epsilon))) \right] \quad (3.4)$$

with the coefficient κ is strictly of $\mathcal{O}(1)$ no matter how ν_i is relative to ϵ . So whether the higher-order corrections to y_{i*} in (3.4) are substantial depend on the finite quantities $\mathcal{P}(\nu_i, \epsilon)$ and $\mathcal{Q}(\nu_i, \epsilon)$.

The $\mathcal{O}(\lambda^4)$ corrections to $\gamma_X(\nu_i, \epsilon)$ gives rise to,

$$-\epsilon + \pi^2 \lambda_*^2 - 2\pi^2 \lambda_*^4 \sum_i \frac{1}{\nu_i} \left[\pi^2 c_i^2 \left(1 - 16 \frac{\epsilon \nu_i}{\nu_i^2 - 4\epsilon^2} \right) \mathcal{I}(\nu_i, \epsilon) - \frac{\nu_i}{\epsilon} \left(\frac{3 - \epsilon}{2} - 2\mathcal{T}(\epsilon) \right) \right] = 0 \quad (3.5)$$

Substitute the leading order approximation $\lambda_*^2 \simeq \frac{\epsilon}{\pi^2}$ into terms of order $\mathcal{O}(\lambda^4)$ in (3.5) gives rise to

$$\lambda_*^2 \simeq -\frac{\epsilon}{\pi^2} + \frac{1}{\pi^2} \mathcal{O} \left(\frac{\epsilon^2 c_i^2}{\nu_i} \right) + \frac{\mathcal{T}(\epsilon)}{\pi^2} \mathcal{O}(\epsilon) \quad (3.6)$$

it is clear to notice that the higher-order corrections can be substantial for determining the fixed point coupling λ_* when $c_i < \nu_i$ and even dominate over the order of $\mathcal{O}(\lambda^2 y_i)$ when $c_i \ll \nu_i$. In the region of small c_i , $c_i \ll \nu_i$, the $\mathcal{O}(\lambda^4)$ correction is substantial for determining the fixed point coupling λ_* .

Now we calculate the anomalous dimensions of operators imposed of L_i , $X^\dagger X$ and their mixing, which can be read from the τ matrix defined as $\tau \equiv \partial_{(y_i, \lambda)} \beta_{(y_i, \lambda)} |_{y_{i*}, \lambda_*}$. By using (3.2) we obtain,

$$\tau = \begin{pmatrix} \nu_i \delta_{ij} & -c_i \lambda_* - 4 \sum_i (4\epsilon + \nu_i) b_{2i} \lambda_*^3 \\ -4\pi^4 \sum_i c_i \mathcal{I}(\nu_i, \epsilon) \lambda_*^3 & 2\epsilon (1 + \frac{5\epsilon^2}{\pi^4} a_2) \end{pmatrix} \quad (3.7)$$

The deviation of the eigenvalues δ of this τ matrix to the case without $\mathcal{O}(\lambda^4)$ effects can be more clearly seen after we make a 2ϵ shift in τ , which is a operation useful for us to directly compare the value of δ_{min} with [10],

$$\delta\tau = \begin{pmatrix} (\nu_i - 2\epsilon) \delta_{ij} & -c_i \lambda_* - 4 \sum_i (4\epsilon + \nu_i) b_{2i} \lambda_*^3 \\ -4\pi^4 \sum_i c_i \mathcal{I}(\nu_i, \epsilon) \lambda_*^3 & \frac{10\epsilon^3}{\pi^4} a_2 \end{pmatrix} \quad (3.8)$$

The point is that all the diagonal elements aren't zero, which remain after a similarity transformation to τ . So whether there exists such a negative δ is not obvious anymore. In general it is quite difficult to obtain the eigenvalues δ without given the information about relative values of ν_i and ϵ . We divide this task into a few cases. The first, also trivial case is $\nu_i \ll \epsilon \ll 1$, in which there are already some L_i with dimension smaller than $2\Delta_X$. The other cases $\epsilon \ll \nu_i \ll 1$ and $\epsilon \sim \nu_i \ll 1$ are of more interest to us.

3.1 $\epsilon \ll \nu_i \ll 1$

Now we address the simplification for the functionals as defined in appendix A in the region $\epsilon \ll \nu_i \ll 1$. Each integral variable X_i^+ in these functionals are evaluated in the region $|X_i^+| > \frac{1}{\Lambda}$, with Λ the cut-off scale introduced in (A.4), and integral over Grassmann variables is equivalent to performing derivative over them. For functional $\mathcal{I}(\nu_i, \epsilon)$ (A.16), performing the integral gives us,

$$\mathcal{I}(\nu_i, \epsilon)|_{\nu_i \ll 1, \epsilon \ll 1} \simeq 1 + \mathcal{O}(\epsilon, \nu_i) \quad (3.9)$$

Similar operation can be applied to $\mathcal{P}(\nu_i, \epsilon)$ functional, which explicitly reads,

$$\mathcal{P}(\nu_i, \epsilon)|_{\nu_i \ll 1, \epsilon \ll 1} \simeq \frac{2\epsilon - \nu_i}{2\epsilon + \nu_i} + \mathcal{O}(\epsilon, \nu_i) \simeq -1 + \mathcal{O}(\epsilon, \nu_i) \quad (3.10)$$

after setting $\nu_i = 0$ and replacing $2\epsilon \rightarrow 2\epsilon + \nu_i$. The functionals $\mathcal{Q}(\nu_i, \epsilon)$ and $\mathcal{T}(\epsilon)$ are three-dimensional integrals, thus more involved than $\mathcal{I}(\nu_i, \epsilon)$ and $\mathcal{P}(\nu_i, \epsilon)$. For this case one can integrate over one variable, then follow the similar operation for the two-dimensional integral. At leading order, we find

$$\begin{aligned} \mathcal{Q}(\nu_i, \epsilon)|_{\nu_i \ll 1, \epsilon \ll 1} &\simeq \left(\frac{2\epsilon - \nu_i}{2\epsilon + \nu_i} \right) \epsilon \Gamma(2\epsilon) + \dots = -1 + \mathcal{O}(\epsilon, \nu_i) \\ \mathcal{T}(\nu_i, \epsilon)|_{\nu_i \ll 1, \epsilon \ll 1} &\simeq +\epsilon^2 [\Gamma(2\epsilon) + \dots] = +\mathcal{O}(\epsilon) + \dots \end{aligned} \quad (3.11)$$

where we have ignored the higher-order terms. The coefficients at the leading order, related to the complicated Hypergeometric function ${}_2F_1(1, m - 2\epsilon, 1 + 2\epsilon, -1)$ (with integer m), are finite and not shown explicitly. We will see the approximations (3.9)-(3.11) are sufficient to illustrate the modification to anomalous dimensions of \mathcal{S}_i .

Substitute (3.9)- (3.11) into (3.1), one can substantially simplify a_2 and b_{2i} . Doing so, we obtain the leading-order approximation to the matrix $\delta\tau$ in (3.8) under the limit $\epsilon \ll \nu_i \ll 1$,

$$\delta\tau = \begin{pmatrix} \nu_i \delta_{ij} & -c_i \lambda_* \left[1 + \frac{1}{\epsilon} (3 + 4\pi^2) \right] \\ -4\pi^4 \sum_i c_i \lambda_*^3 & -\frac{20\epsilon}{\pi^2} \mathcal{T}(\epsilon) \end{pmatrix} \quad (3.12)$$

Put the values of couplings at NLO back into (3.12), the characteristic equation of δ is found to be,

$$(\nu_i \delta_{ij} - \delta) \left(-\frac{20\epsilon}{\pi^2} \mathcal{T}(\epsilon) - \delta \right) - 4\pi^2(3 + 4\pi^2) c_i^2 \epsilon = 0 \quad (3.13)$$

Together with the small $c_i \ll \nu_i$ condition assumed through out this section, we notice that the last constant term in (3.13) is actually small compared with $\nu_i \epsilon$ if c_i is below the critical value $c_{i*} \simeq \sqrt{\nu_i \epsilon}$, which implies that the minimal value of δ is of order ϵ^2 ,

$$\delta_{min} \simeq -\frac{20\epsilon}{\pi^2} \mathcal{T}(\epsilon) + \mathcal{O}(\epsilon^3) < 0 \quad (3.14)$$

One thing happens when c_i is above the critical value c_{i*} . The last term dominate conversely, which modified the (3.13) as,

$$\delta_{min} \simeq -\pi^2(3 + 4\pi^2) \frac{c_i^2 \epsilon}{\nu_i} \simeq -\pi^2(3 + 4\pi^2) \left(\frac{c_i}{c_{i*}} \right)^2 \epsilon^2 < 0 \quad (3.15)$$

3.2 $\nu_i \sim \epsilon \ll 1$

Since it is quite natural to expect that ν_i is of $\mathcal{O}(\epsilon)$ or higher powers of ϵ in perturbative CFT, a number of \mathcal{P}_2 theories can be covered in this limit. Now we address the question that whether the statement in the previous discussion can be generalized to this particular situation. At first, a_2 and b_{2i} take the approximation ²,

$$\begin{aligned} a_2 &= -\frac{2\pi^2}{\epsilon^2} \mathcal{T}(\epsilon) \\ b_{2i} &= -\frac{3\pi^2 c_i}{4\epsilon(\nu_i + 2\epsilon)} \end{aligned} \quad (3.16)$$

Substitute these values into (3.8), we obtain

$$\delta\tau = \begin{pmatrix} (\nu_i - 2\epsilon)\delta_{ij} & -c_i \lambda_* [1 + \mathcal{O}(\epsilon^{-1}\lambda^2)] \\ -4\pi^4 \sum_i c_i \lambda_*^3 & -\frac{20\epsilon}{\pi^2} \mathcal{T}(\epsilon) \end{pmatrix} = \begin{pmatrix} (\nu_i - 2\epsilon)\delta_{ij} & \mathcal{O}(c_i^{\frac{1}{2}}) \\ \mathcal{O}(c_i \epsilon^{\frac{3}{2}}) & -\frac{20\epsilon}{\pi^2} \mathcal{T}(\epsilon) \end{pmatrix} \quad (3.17)$$

Drop the off-diagonal elements in above matrix by using the relation $c_i \ll \nu_i \sim \epsilon$, we arrive at the conclusion that the statement is also true in the region.

In summary, if $c_i \ll \nu_i \ll 1$ is indeed produced given a \mathcal{P}_2 theory, then we can conclude that the bound on the anomalous dimension of \mathcal{S}_i as conjectured in the literature

²Note that a_2 has a pole at $\nu_i = 2\epsilon$. Here, we assume ν_i is not equal to 2ϵ for simplification.

is still valid at NNLO , no matter the relative values of ϵ and ν_i . Therefore, the validity of this conjecture is directly transferred to examine these conditions in \mathcal{P}_2 theory ³.

4 SCFTs at $\mathcal{O}(y^2)$

The $\mathcal{O}(y^2)$ corrections dominate over $\mathcal{O}(\lambda^4)$ when $c_i \gg \nu_i$. The investigation of bounds on c_i can be found in [17, 18]. Instead of calculating the wave-function renormalization and beta function as in appendix A, one must consider L_i operators. But this task can not be precisely achieved without knowing the explicit form of L_i (for example L_i are composite operators). The $\mathcal{O}(y^2)$ effects can only be analyzed either in a specific \mathcal{P}_1 theory or in certain approximations.

4.1 BZ Theory As an illustration

One might wonder which \mathcal{P}_1 theory can provide such kind of condition. Actually, given a special choice of the flavor number N_f and rank of gauge group N_c , the BZ theory [7] could be a simple realization. It is classified in [10] that $L = bTr(Q^\dagger Q + \tilde{Q}^\dagger \tilde{Q})$ in the BZ theory, with Q_i being the chiral matter superfields. Under the large N limit with $\frac{N_f}{3N_c} = 1 + \epsilon$ and normalizations taken in Ref [10] , it is found that $c_L = \sqrt{\frac{2}{N_f N_c}}$ and $\nu_L \simeq 3\epsilon^2$. Impose the constraint $c_i \ll \nu_i$, we find $\epsilon^2 \ll \frac{1}{N_c}$. Take the perturbative condition $y \simeq \frac{c_i}{\nu_i} \lambda^2 \simeq \frac{c_i \epsilon}{\nu_i}$ into account , we obtain $\epsilon \gg \frac{1}{N_c}$ for consistency. So if ϵ which can be considered as an input parameter is left to be in the narrow window

$$\frac{1}{N_c} \ll \epsilon \ll \frac{1}{\sqrt{N_c}} \quad (4.1)$$

then higher-order corrections in this BZ theory arising from $\mathcal{O}(y^2)$ indeed dominate over $\mathcal{O}(\lambda^4)$.

To estimate the $\mathcal{O}(y^2)$ corrections to the matrix of anomalous dimensions at NLO [10],

$$\tau = \begin{pmatrix} \nu_L \simeq 3\epsilon^2 & -\frac{3\epsilon^2}{N_c^2} \\ -\frac{4}{3}\epsilon & 2\epsilon \end{pmatrix} \quad (4.2)$$

³We want to remind the reader that naively this statement can not be directly applied to BZ theory with large N limit. However, in BZ theory $\nu_i \simeq \mathcal{O}(\epsilon^2)$ [10], which actually suggests some of anomalous dimension of L_i is already smaller than that of X . This statement is trivially satisfied in this situation.

one must consider the higher-order terms in the anomalous dimensions of Q and X , especially those unsuppressed by $1/N$. From [19] (see also [10]) we obtain,

$$\delta\gamma_Q(\hat{g}, \lambda) = \frac{2-\epsilon}{1+\epsilon}\hat{g}^2 + \mathcal{O}(\hat{g}^2/N_c^2), \quad \delta\gamma_X(\hat{g}, \lambda) \simeq \frac{\hat{g}\hat{\lambda}}{N_c^2} + \mathcal{O}(\hat{g}\hat{\lambda}/N_c^2) \quad (4.3)$$

where $\hat{g} = \frac{N_c g^2}{16\pi^2}$. Substituting (4.3) into the τ matrix leads to correction to (4.2),

$$\delta\tau = \begin{pmatrix} -12\hat{g}_*^3 & 0 \\ \frac{16}{3}\epsilon^2 & \frac{4\epsilon^2}{N_c^2} \end{pmatrix} = \begin{pmatrix} \mathcal{O}(\epsilon^3) & 0 \\ \mathcal{O}(\epsilon^2) & \mathcal{O}(\epsilon^3) \sim \mathcal{O}(\epsilon^4) \end{pmatrix} \ll \tau \quad (4.4)$$

by using the constraint (4.1). Unlike the situation in the previous section, each matrix element is smaller compared with those at NLO in this case. This suggests that the ability to affect the sign of δ_{min} coming from $\mathcal{O}(y^2)$ is weaker than $\mathcal{O}(\lambda^4)$.

4.2 Analysis of OPE

The simple example of BZ theory in the previous discussion provides us an intuition that the $\mathcal{O}(y^2)$ corrections are probably negligible under the assumptions taken by us in the setup. Now we address this issue by analyzing the OPEs in this case. The estimate of $\mathcal{O}(y^2)$ effects involved the calculations of coefficients a_{2ij} and b_{2ijk} . The possible combinations that contribute to coefficient b_{2ijk} are *null* due to the fact that all of the coefficients at $\mathcal{O}(y)$ vanish in (2.4) and (2.5). For a_{2ij} , by using the results in (2.4) and (2.5) all the combinations of operators do not contribute, which gives us

$$a_{2ij} = 0, \quad \text{and} \quad b_{2ijk} = 0 \quad (4.5)$$

In summary, the NNLO corrections due to $\mathcal{O}(y^2)$ are actually null. The statement in the previous section holds also in the region of $c_i \gg \nu_i$ (but still on the realm of perturbative field theory).

What about the higher-order terms involved y_i couplings. The vanishing contributions both at NLO and NNLO indicates that the contributions arising from y_i beyond NLO do not exist, i.e, the coefficients in powers of $y_i^n \lambda^m$ ($n = 2, 3, \dots$, $m = 0, 1, \dots$) are *exactly* zero. In general, these operators are related to the following OPEs,

$$\begin{aligned} & X^\dagger(z_1^-) L_i(x_2, \theta_2, \bar{\theta}_2) \\ & X(z_1^+) L_i(x_2, \theta_2, \bar{\theta}_2) \\ & L_i(x_1, \theta_1, \bar{\theta}_1) L_j(x_2, \theta_2, \bar{\theta}_2) \end{aligned} \quad (4.6)$$

To determine the OPEs in (4.6), we use a crucial observation in our setup. At first, the primary operators \mathcal{S}_i are composed of primary operators L_i and $X^\dagger X$ because of the interaction mediated by λ . This implies that \mathcal{S}_i can be generally expressed as⁴,

$$\mathcal{S}_i(x) = \frac{\cos \alpha_i}{|x|^{\tilde{\Delta}_i - \Delta_i}} L_i - \frac{\sin \beta_i}{|x|^{\tilde{\Delta}_i - \Delta_{X^\dagger X}}} X^\dagger X(x) + \dots \quad (4.7)$$

Angle α_i and β_i are introduced to represent the mixings. Here we refer $\tilde{\Delta}_i$ to the scaling dimension of \mathcal{S}_i . What are ignored in (4.7) are irrelevant for our purpose. Define the d_i as the OPE coefficient in three-point correlator :

$$\langle X^\dagger(z_2^-, \bar{\theta}_2) X(z_1^+, \theta_1) \mathcal{S}_i(x_3, \theta_3, \bar{\theta}_3) \rangle = \frac{d_i}{(X_{21}^+)^{2\Delta_X - \tilde{\Delta}_i} (X_{23}^+)^{\tilde{\Delta}_i} (X_{31}^+)^{\tilde{\Delta}_i}} \quad (4.8)$$

We can subtract the OPEs in (4.6) by the OPEs of \mathcal{S}_i s. From (4.8) we obtain the two-point OPEs:

$$\begin{aligned} \mathcal{S}(x_3, \theta_3, \bar{\theta}_3) X^\dagger(z_2^-, \bar{\theta}_2) &\rightarrow \frac{d_i}{(X_{23}^+)^{\tilde{\Delta}_i}} X^\dagger(z_2^-, \bar{\theta}_2) + \dots \\ \mathcal{S}(x_3, \theta_3, \bar{\theta}_3) X(z_1^+, \theta_1) &\rightarrow \frac{d_i}{(X_{31}^+)^{\tilde{\Delta}_i}} X(z_1^+, \theta_1) + \dots \end{aligned} \quad (4.9)$$

Now we derive the OPEs in (4.6). From (4.7) we obtain,

$$L_i \simeq \frac{\cos \alpha_i}{|x|^{\Delta_i - \tilde{\Delta}_i}} \mathcal{S}_i(x) + \frac{\sin \beta_i}{|x|^{\Delta_i - 2}} X^\dagger X(x) + \dots \quad (4.10)$$

Consequently, the OPEs (4.6) can be derived in terms of (4.10), (4.8) and (4.9),

$$L_i(x_1, \theta_1, \bar{\theta}_1) L_j(x_2, \theta_2, \bar{\theta}_2) \rightarrow \frac{\sin \beta_i \sin \beta_j}{|x_1|^{\Delta_i - 2} |x_2|^{\Delta_j - 2} (X_{21}^+)^2} X^\dagger X(x_1, \theta_1, \bar{\theta}_2) + \dots \quad (4.11)$$

and

$$\begin{aligned} X^\dagger X(x_1, \theta_1, \bar{\theta}_1) L_i(x_2, \theta_2, \bar{\theta}_2) L_j(x_3, \theta_3, \bar{\theta}_3) &\rightarrow \\ \frac{1}{(X_{12}^+)^{\tilde{\Delta}_i} (X_{31}^+)^{\tilde{\Delta}_i} |x_2|^{\Delta_i - \tilde{\Delta}_i} |x_3|^{\Delta_j - \tilde{\Delta}_i}} &[\cos \alpha_i \cos \alpha_j d_i d_j X^\dagger X(x_1, \theta_1, \bar{\theta}_2) + \dots] \end{aligned} \quad (4.12)$$

where in the second line in (4.12) refer to similar structure of $X^\dagger X$.

⁴We understand this expression is not exact from the viewpoint of superconformal symmetries, but it indeed captures the main property of scaling dimension relevance, which is the central concern of this note. Also note that operators composed of (super)derivative over operators on the RHS of (4.7) are not permitted.

Consider the coefficient a_{3ij} that appears in $a_{3ij}y_iy_j\lambda^2$ as an example at the next-to-NNLO. The combinations arising from multiple $X^\dagger X$ themselves do not contribute, with only those possibilities in (4.6) left. Substitute (4.12) and (4.11) into the operators that contribute to $a_{3ij}y_iy_j\lambda^2$, we find that both of them vanish due to the residual Grassmann integrals. We conclude that the claim on null contribution coming from y_i coupling beyond NLO still holds.

5 Conclusions

In this note we study the effects of NNLO corrections on the conjecture that $\delta_{min} < 0$, in the context of perturbative CFT. As we have emphasized, despite smaller than NLO ones, the NNLO corrections are important and even substantial in some circumstances. In particular, the modifications to the vanishing matrix elements of anomalous dimension at NLO can directly affect the sign of δ_{min} , although they don't substantially modify the values of fixed points couplings λ_* and y_{i*} .

The main results include:

1. In the region of $c_i \ll \nu_i \ll 1$ in a \mathcal{P}_2 theory as defined in the introduction, the bound on the anomalous dimension of \mathcal{S}_i as conjectured in the literature is still valid at NNLO, no matter the relative values of ϵ and ν_i .
2. In the region of $c_i \gg \nu_i$ the NNLO corrections due to $\mathcal{O}(y^2)$ effects are actually null. the conjecture still holds.
3. The null contribution arising from y_i couplings beyond NLO *exactly* remains.

There are a few points that deserve further investigation. For instance, one can examine the conjecture in background of strongly coupled SCFTs via method of ADS/CFT. Throughout this note, we have not addressed the possibility that there are residual global symmetries after imposing the deformation, it would be also interesting to discuss this issue in the further.

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A OPEs and $\mathcal{O}(\lambda^4)$ Effects

In superspace, the two-point functions for $\mathcal{O}\mathcal{O}^\dagger$, XX^\dagger and three-point function for $L\mathcal{O}\mathcal{O}^\dagger$ are given by [3, 10],

$$\begin{aligned} \langle \mathcal{O}(z_1^+, \theta_1) \mathcal{O}^\dagger(z_2^-, \bar{\theta}_2) \rangle &= \frac{1}{(X_{21}^+)^{2(2-\epsilon)}} \\ \langle X(z_1^+, \theta_1) X^\dagger(z_2^-, \bar{\theta}_2) \rangle &= \frac{1}{(X_{21}^+)^2} \\ \langle \mathcal{O}(z_1^+, \theta_1) \mathcal{O}^\dagger(z_2^-, \bar{\theta}_2) L(x_3, \theta_3, \bar{\theta}_3) \rangle &= \frac{c_i}{(X_{21}^+)^{2-2\epsilon-\nu_i} (X_{23}^+)^{2+\nu_i} (X_{31}^+)^{2+\nu_i}} \end{aligned} \quad (\text{A.1})$$

where $X_{ij}^+ = z_i^- - z_j^+ + 2i\theta_j\sigma\bar{\theta}_i$ is a supertranslation invariant interval. Here $z^\pm = x \pm i\theta\sigma\bar{\theta}$. We also need the following superspace OPEs that can be derived from (A.1),

$$\begin{aligned} \mathcal{O}^\dagger(z_2^-, \bar{\theta}_2) \mathcal{O}(z_1^+, \theta_1) &\rightarrow \frac{1}{(X_{21}^+)^{2(2-\epsilon)}} + \frac{c_i}{(X_{21}^+)^{2-2\epsilon-\nu_i}} L_i + \dots \\ X^\dagger(z_2^-, \bar{\theta}_2) X(z_1^+, \theta_1) &= \frac{1}{(X_{21}^+)^2} + X^\dagger X(x_1, \theta_1, \bar{\theta}_2) + \dots \end{aligned} \quad (\text{A.2})$$

and

$$\begin{aligned} L(x_3, \theta_3, \bar{\theta}_3) \mathcal{O}^\dagger(z_2^-, \bar{\theta}_2) &= \frac{c_i}{(X_{23}^+)^{2+\nu_i}} \mathcal{O}^\dagger(z_2^-, \bar{\theta}_2) + \dots \\ L(x_3, \theta_3, \bar{\theta}_3) \mathcal{O}(z_1^+, \theta_1) &= \frac{c_i}{(X_{31}^+)^{2+\nu_i}} \mathcal{O}(z_1^+, \theta_1) + \dots \end{aligned} \quad (\text{A.3})$$

The terms ignored in (A.2) and (A.3) are superconformal descendant, which are irrelevant for our calculations of beta function.

The terms involved in $\mathcal{O}(\lambda^4)$ wave-function renormalization can be read from (2.1) and (2.3),

$$\begin{aligned} \frac{1}{4\pi^2} \int d^4x d^4\theta (1 + a_1\lambda^2 + a_2\lambda^4 + \dots) X^\dagger X + \int d^4x d^4\theta (y_i + b_{1i}\lambda^2 + b_{2i}\lambda^4 + \dots) \lambda^{-\nu_i} L_i \\ + \frac{\lambda}{2\pi} \left(\int d^4z_2^+ d\theta_2^2 \Lambda^\epsilon \mathcal{O} X(z_2^+, \theta_2) + \int d^4z_1^- d\bar{\theta}_1^2 \Lambda^\epsilon \mathcal{O}^\dagger X^\dagger(z_1^-, \bar{\theta}_1) \right) \end{aligned} \quad (\text{A.4})$$

Evaluating (A.4) we obtain the counter terms of order $\mathcal{O}(\lambda^4)$,

$$\begin{aligned}
& \lambda^4 \left[\frac{a_2}{4\pi^2} \int d^4x d^4\theta X^\dagger X(x, \theta, \bar{\theta}) + b_{2i} \lambda^{-\nu_i} \int d^4x d^4\theta L_i \right. \\
& + \frac{b_{2i}}{4\pi^2} \Lambda^{-\nu_i} \int d^4x_1 d^4x_2 d^4\theta_1 d^4\theta_2 X^\dagger X(x_2, \theta_2, \bar{\theta}_2) L_i(x_1, \theta_1, \bar{\theta}_1) \\
& + \frac{1}{(4\pi^2)^2} \int d^4x_1 d^4x_2 d^4\theta_1 d^4\theta_2 X^\dagger X(x_2, \theta_2, \bar{\theta}_2) X^\dagger X(x_1, \theta_1, \bar{\theta}_1) \\
& + \frac{a_1}{(4\pi^2)^2} \Lambda^{2\epsilon} \int d^4z_1^- d^4z_2^+ d^4x_3 d^2\bar{\theta}_1 d^2\theta_2 d^4\theta_3 \mathcal{O}^\dagger X^\dagger(z_1^-, \bar{\theta}_1) \mathcal{O}X(z_2^+, \theta_2) X^\dagger X(x_3, \theta_3, \bar{\theta}_3) \\
& + \frac{b_{1i}}{4\pi^2} \Lambda^{2\epsilon-\nu_i} \int d^4z_1^- d^4z_2^+ d^4x_3 d^2\bar{\theta}_1 d^2\theta_2 d^4\theta_3 \mathcal{O}^\dagger X^\dagger(z_1^-, \bar{\theta}_1) \mathcal{O}X(z_2^+, \theta_2) L_i(x_3, \theta_3, \bar{\theta}_3) \\
& + \frac{1}{(4\pi^2)^2} \Lambda^{4\epsilon} \int d^4z_1^- d^4z_2^+ d^4z_3^- d^4z_4^+ d^2\bar{\theta}_1 d^2\theta_2 d^2\bar{\theta}_3 d^2\theta_4 \mathcal{O}^\dagger X^\dagger(z_1^-, \bar{\theta}_1) \mathcal{O}X(z_2^+, \theta_2) \\
& \times \left. \mathcal{O}^\dagger X^\dagger(z_3^-, \bar{\theta}_3) \mathcal{O}X(z_4^+, \theta_4) \right] \tag{A.5}
\end{aligned}$$

which gives us,

$$\begin{aligned}
- b_{2i} &= \left[\frac{a_1}{(4\pi^2)^2} \Lambda^{2\epsilon+\nu_i} \int d^4z_1^- d^4x_3 d^4\theta_3 \frac{c_i}{(X_{32}^+)^2 (X_{13}^+)^2 (X_{12}^+)^{2-2\epsilon-\nu_i}} \right. \\
& + \frac{b_{1i}}{4\pi^2} \Lambda^{2\epsilon} \int d^4z_1^- d^4z_2^+ d^2\bar{\theta}_1 d^2\theta_2 \frac{1}{(X_{12}^+)^{6-2\epsilon}} \\
& \left. + 4 \times \frac{c_i}{(4\pi^2)^2} \Lambda^{4\epsilon+\nu_i} \int \frac{d^4z_1^- d^4z_2^+ d^4z_3^- d^2\theta_2 d^2\bar{\theta}_3}{(X_{12}^+)^2 (X_{34}^+)^2 (X_{14}^+)^{2-2\epsilon-\nu_i} (X_{32}^+)^{2(2-\epsilon)}} \right] \tag{A.6}
\end{aligned}$$

from the last three terms in (A.5) and OPEs given in (A.2). The factor 4 in the last line in (A.6) counts the four symmetric permutations. Performing the intergral of the first line in (A.6) gives,

$$\frac{a_1 c_i}{(4\pi^2)^2} \Lambda^{2\epsilon+\nu_i} \int \frac{d^4X_{13}^+ d^4X_{32} d^4\theta_{32}}{(X_{32}^+)^2 (X_{13}^+)^2 (X_{12}^+)^{2-2\epsilon-\nu_i}} \equiv \frac{1}{2(\nu_i - 2\epsilon)} a_1 c_i \mathcal{P}(\nu_i, \epsilon) \tag{A.7}$$

with

$$\begin{aligned}
\mathcal{P}(\nu_i, \epsilon) &= \frac{(\nu_i - 2\epsilon)}{8\pi^4} \Lambda^{2\epsilon+\nu_i} \int \frac{d^4X_{13}^+ d^4X_{32} d^4\theta_{32}}{(X_{32}^+)^2 (X_{13}^+)^2 (X_{12}^+)^{2-2\epsilon-\nu_i}} \\
&= \frac{(\nu_i - 2\epsilon)(3 - 2\epsilon - \nu_i)(2 - 2\epsilon - \nu_i)}{8\pi^4} \Lambda^{2\epsilon+\nu_i} \int \frac{d^4X_{13}^+ d^4X_{32}^+}{(X_{32}^+)^2 (X_{13}^+)^2 (X_{13}^+ + X_{32}^+)^{4-2\epsilon-\nu_i}} \tag{A.8}
\end{aligned}$$

where we have changed the integration variables $z_1^- \rightarrow X_{13}^+$, $x_3 \rightarrow X_{32}^+$, $\theta_3 \rightarrow \theta_{32}$ and $\bar{\theta}_3 \rightarrow \bar{\theta}_{32}$, and use the equality $X_{12}^+ = X_{13}^+ + X_{32}^+ + 2i\theta_{32}\sigma\bar{\theta}_{32}$. The second integral in (A.6)

is equal to,

$$\frac{b_{1i}}{4\pi^2} \Lambda^{2\epsilon} \int \frac{d^4 X_{12}^+ d^4 X_{23}^+ d^2 \bar{\theta}_{12} d^2 \theta_{23}}{(X_{12}^+)^{6-2\epsilon}} = 0 \quad (\text{A.9})$$

after we are free to change the integral variables $z_1^- \rightarrow X_{12}^+$, $z_2^+ \rightarrow X_{23}^+$, $\bar{\theta}_1 \rightarrow \bar{\theta}_{12}$ and $\theta_2 \rightarrow \theta_{23}$. The last integral in (A.6) can be reexpressed as,

$$4 \times \frac{c_i}{(4\pi^2)^2} \Lambda^{4\epsilon+\nu_i} \int \frac{d^4 z_1^- d^4 z_2^+ d^4 z_3^- d^2 \theta_2 d^2 \bar{\theta}_3}{(X_{12}^+)^2 (X_{34}^+)^2 (X_{14}^+)^{2-2\epsilon-\nu_i} (X_{32}^+)^{2(2-\epsilon)}} \equiv \frac{\pi^2 c_i}{2\epsilon(\nu_i - 2\epsilon)} \mathcal{Q}(\nu_i, \epsilon) \quad (\text{A.10})$$

with

$$\begin{aligned} \mathcal{Q}(\nu_i, \epsilon) &= \frac{\epsilon(\nu_i - 2\epsilon)}{2\pi^6} \Lambda^{4\epsilon+\nu_i} \int \frac{d^4 z_1^- d^4 z_2^+ d^4 z_3^- d^2 \theta_2 d^2 \bar{\theta}_3}{(X_{12}^+)^2 (X_{34}^+)^2 (X_{14}^+)^{2-2\epsilon-\nu_i} (X_{32}^+)^{2(2-\epsilon)}} \\ &= \frac{\epsilon(\nu_i - 2\epsilon)}{2\pi^6} \Lambda^{4\epsilon+\nu_i} \int \frac{d^4 X_{12}^+ d^4 X_{32}^+ d^4 X_{34}^+ d^2 \theta_{42} d^2 \bar{\theta}_{13}}{(X_{12}^+)^2 (X_{34}^+)^2 (X_{12}^+ - X_{32}^+ + X_{34}^+ + 2i\theta_{42}\sigma\bar{\theta}_{13})^{2-2\epsilon-\nu_i} (X_{32}^+)^{2(2-\epsilon)}} \\ &= \frac{\epsilon(\nu_i - 2\epsilon)(3 - 2\epsilon - \nu_i)(2 - 2\epsilon - \nu_i)}{2\pi^6} \Lambda^{4\epsilon+\nu_i} \\ &\quad \times \int \frac{d^4 X_{12}^+ d^4 X_{32}^+ d^4 X_{34}^+}{(X_{12}^+)^2 (X_{34}^+)^2 (X_{12}^+ - X_{32}^+ + X_{34}^+)^{4-2\epsilon-\nu_i} (X_{32}^+)^{2(2-\epsilon)}} \end{aligned} \quad (\text{A.11})$$

after we change the integral variables $z_1^- \rightarrow X_{12}^+$, $z_2^+ \rightarrow -X_{32}^+$, $z_3^- \rightarrow X_{34}^+$, $\theta_2 \rightarrow -\theta_{42}$ and $\bar{\theta}_3 \rightarrow -\bar{\theta}_{13}$ and use the equality $X_{14}^+ = X_{12}^+ - X_{32}^+ + X_{34}^+ + 2i\theta_{42}\sigma\bar{\theta}_{13}$.

Collect the results in (A.10), (A.9) and (A.7), we have the final result about b_{2i} ,

$$b_{2i} = -\frac{\pi^2 c_i}{2\epsilon(\nu_i - 2\epsilon)} [\mathcal{P}(\nu_i, \epsilon) + \mathcal{Q}(\nu_i, \epsilon)] \quad (\text{A.12})$$

Similarly, the methods can be applied to calculating a_2 in (A.5). Doing so gives us the final result of a_2 ,

$$\begin{aligned} -\frac{a_2}{4\pi^2} &= \frac{a_1}{(4\pi^2)^2} \Lambda^{2\epsilon} \left[\left(\int \frac{d^4 z_1^- d^4 z_2^+ d^2 \theta_2 d^2 \bar{\theta}_3}{(X_{32}^+)^2 (X_{21}^+)^{2(2-\epsilon)}} + \text{permutations} \right) + \int \frac{d^4 z_1^- d^4 z_2^+ d^2 \bar{\theta}_1 d^2 \theta_2}{(X_{21}^+)^{6-2\epsilon}} \right] \\ &+ \frac{b_{1i} c_i}{4\pi^2} \Lambda^{2\epsilon-\nu_i} \int \frac{d^4 z_1^- d^4 x_3 d^4 \theta_3}{(X_{12}^+)^{2-2\epsilon-\nu_i} (X_{13}^+)^{2+\nu_i} (X_{32}^+)^{2+\nu_i}} \\ &+ 4 \times \frac{1}{(4\pi^2)^2} \Lambda^{4\epsilon} \int \frac{d^4 z_1^- d^4 z_3^- d^4 z_4^+ d^2 \bar{\theta}_3 d^2 \theta_4}{(X_{32}^+)^{2(2-\epsilon)} (X_{14}^+)^{2(2-\epsilon)} (X_{34}^+)^2} \end{aligned} \quad (\text{A.13})$$

The first integral in the first line of (A.13) do not contributes, while the second integral is the similar to (A.9),

$$\frac{a_1}{(4\pi^2)^2} \Lambda^{2\epsilon} \int \frac{d^4 z_1^- d^4 z_2^+ d^2 \bar{\theta}_1 d^2 \theta_2}{(X_{12}^+)^{6-2\epsilon}} = 0 \quad (\text{A.14})$$

The second one can be simplified by introducing the $\mathcal{I}(\nu_i, \epsilon)$ function as in [10], which results in,

$$b_{1i}c_i\Lambda^{2\epsilon-\nu_i}\int\frac{d^4z_1^-d^4x_3d^4\theta_3}{(X_{12}^+)^{2-2\epsilon-\nu_i}(X_{13}^+)^{2+\nu_i}(X_{32}^+)^{2+\nu_i}}\equiv-8\pi^4\frac{b_{1i}c_i}{\nu_i-2\epsilon}\mathcal{I}(\nu_i,\epsilon)\quad(\text{A.15})$$

with

$$\mathcal{I}(\nu_i,\epsilon)=-\frac{(\nu_i-2\epsilon)(3-2\epsilon-\nu_i)(2-2\epsilon-\nu_i)}{8\pi^4}\Lambda^{2\epsilon-\nu_i}\int\frac{d^4X_{23}^+d^4X_{31}^+}{(X_{23}^++X_{31}^+)^{4-2\epsilon-2\nu_i}(X_{23}^+)^{2+\nu_i}(X_{31}^+)^{2+\nu_i}}\quad(\text{A.16})$$

The last integral in (A.13)

$$\begin{aligned} 4 &\times \frac{1}{(4\pi^2)^2}\Lambda^{4\epsilon}\int\frac{d^4z_1^-d^4z_3^-d^4z_4^+d^2\bar{\theta}_3d^2\theta_4}{(X_{32}^+)^{2(2-\epsilon)}(X_{14}^+)^{2(2-\epsilon)}(X_{34}^+)^2} \\ &= 4\times\frac{1}{(4\pi^2)^2}\Lambda^{4\epsilon}\int\frac{d^4X_{12}^+d^4X_{34}^+d^4X_{14}^+d^2\bar{\theta}_{13}d^2\theta_{42}}{(X_{34}^+-X_{14}^++X_{12}^++2i\theta_{42}\sigma\bar{\theta}_{13})^{2(2-\epsilon)}(X_{14}^+)^{4-2\epsilon}(X_{34}^+)^2}\equiv\frac{8\pi^6}{(4\pi^2)^2\epsilon^2}\mathcal{T}(\epsilon) \end{aligned}\quad(\text{A.17})$$

with

$$\mathcal{T}(\epsilon)=\frac{\epsilon^2(-2+2\epsilon)(-5+2\epsilon)}{2\pi^6}\Lambda^{4\epsilon}\int\frac{d^4X_{12}^+d^4X_{34}^+d^4X_{14}^+}{(X_{34}^+-X_{14}^++X_{12}^+)^{6-2\epsilon}(X_{34}^+)^2(X_{14}^+)^{2(2-\epsilon)}}\quad(\text{A.18})$$

after we change the integral variables $z_1^- \rightarrow X_{12}^+$, $z_3^- \rightarrow X_{34}^+$, $z_4^+ \rightarrow -X_{14}^+$, $\bar{\theta}_3 \rightarrow -\bar{\theta}_{13}$, $\theta_4 \rightarrow \theta_{42}$, and use the equality $X_{32}^+ = X_{34}^+ - X_{14}^+ + X_{12}^+ + 2i\theta_{42}\sigma\bar{\theta}_{13}$.

Consequently, we get the final expression of (A.13)

$$a_2=16\pi^4\left(\frac{c_i^2}{\nu_i^2-4\epsilon^2}\right)\mathcal{I}(\nu_i,\epsilon)-\frac{2\pi^2}{\epsilon^2}\mathcal{T}(\epsilon)\quad(\text{A.19})$$

With the help of Mathematica, the functionals defined above can be evaluated.

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